

Forcing-dependent dynamics and emergence of helicity in rotating turbulence

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The effects of large scale mechanical forcing on the dynamics of rotating turbulent flows are studied by means of numerical simulations, varying systematically the nature of the mechanical force in time. We demonstrate that the statistically stationary solutions of these flows depend on the nature of the forcing mechanism. Rapidly enough rotating flows with a forcing that has a persistent direction relatively to the axis of rotation bifurcate from a non-helical state to a helical state despite the fact that the forcing is non-helical. We find that the nature of the mechanical force in time and the emergence of helicity have direct implications on the cascade dynamics of these flows, determining the anisotropy in the flow, the energy condensation at large scales and the power-law energy spectra that are consistent with previous findings and phenomenologies under strong and weak-wave turbulent conditions.

Key words:

1. Introduction

The effects of the Coriolis force on a turbulent fluid flow become important at sufficiently high rotation rates altering its dynamics (Tritton 1988). Experiments and simulations reveal that fast rotation renders the flow quasi-two-dimensional (quasi-2D), since fast rotation suppresses the velocity gradients along the axis of rotation as shown by the Taylor-Proudman theorem (Proudman 1916; Taylor 1917). Under such conditions, the flow sustains inertial waves whose frequency is proportional to the rotation rate (Lighthill 1965; Greenspan 1968).

The interplay between inertial waves and eddies in rotating fluids makes the problem of rotating turbulence very rich. Many experimental studies at a wide range of parameters have elucidated the dynamics of such flows (Hopfinger & Heijst 1993; Ruppert-Felsot *et al.* 2005; Davidson *et al.* 2006; Bewley *et al.* 2007; van Bokhoven *et al.* 2009; Moisy *et al.* 2011; Yarom *et al.* 2013; Campagne *et al.* 2014). Many numerical studies have also been carried out on rotating turbulence. The regime for which both the turbulence is fully developed (large Reynolds numbers) and the flow is fast rotating (small Rossby numbers) puts strong restrictions on the scale separation requirements in simulations. Therefore, most of the early numerical investigations were focused on decaying rotating turbulence (Bartello *et al.* 1994; Hossain 1994; Cambon *et al.* 1997; Morinishi *et al.* 2001; Teitelbaum & Mininni 2011; Yoshimatsu *et al.* 2011). More recent studies have been performed on forced rotating turbulence both at large (Yeung & Zhou 1998; Mininni & Pouquet 2010; Mininni *et al.* 2012) and small scales; the latter to study the dynamics of the inverse cascade (Smith *et al.* 1996; Pouquet *et al.* 2013; Deusebio *et al.* 2014).

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The computational costs prevent in general the exhaustive coverage of the parameters space with most numerical studies reaching either large Reynolds numbers and moderate Rossby numbers or moderate Reynolds numbers and small Rossby numbers. Note that these simulations, with the exception of a very recent study which reached steady states and covered extensively a fairly large portion of the parameters space (Alexakis 2015), have not reached statistically stationary solutions because very long integration times are required.

Although these studies were numerous and an adequate portion of the parameters space was covered there are still disparate results in different cases — for example, different power law spectra with $E(k) \propto k^{-5/3}$, k^{-2} and $k^{-5/2}$, supported theoretically by strong and weak-wave turbulence phenomenologies (Kolmogorov 1941; Zhou 1995; Galtier 2003; Pouquet & Mininni 2010). Moreover, a recent investigation showed sensitivity to forcing in that different results for the large scales of a flow that was forced at intermediate scales (Sen *et al.* 2012) when energy was injected exclusively to the quasi-2D component of the flow and when it was injected solely to the inertial waves. The injection of helicity into the flow has also shown alterations on the behaviour of the cascade (Pouquet & Mininni 2010). Forcing dependent dynamics have been observed in various other systems such as in two-dimensional (2D) turbulence (Bracco & McWilliams 2010; Boffetta & Ecke 2012), in beta-plane turbulence (Maltrud & Vallis 1991) and in magnetohydrodynamic turbulence (Dallas & Alexakis 2015).

The present work focuses on the effects of the mechanical force on the dynamics of rotating flows by means of numerical simulations, varying systematically the memory time scale of the mechanical force (i.e. the time scale of which the phases of the force are randomised). The behaviour of different mechanical forcing mechanisms on the flows is also considered for different rotation rates. To the best of the authors' knowledge this is the first study of forced rotating flows in the steady state regime where the effects of a large scale external force on the dynamics are studied extensively.

The paper is structured as follows. All the necessary details of the formulation of our direct numerical simulations (DNS) of forced rotating turbulence are provided in Sec. 2. Section 3 analyses the dynamics of the flows with different memory time scales of the forcing mechanism for a given Rossby number. Here, we also focus on the spontaneous emergence of helicity in our flows and its influence on the anisotropy and on the spectral dynamics. In Sec. 4 we describe the Rossby number dependence on flows with different type of forcing mechanisms and we justify the spontaneous mirror-symmetry breaking in our flows even though net helicity is not injected directly. Finally, in Sec. 5, we conclude by summarising our findings and we discuss the implications of our work.

2. Numerical Simulations

In this study, we consider the three-dimensional (3D) incompressible Navier-Stokes equations in a rotating frame of reference

$$\partial_t \mathbf{u} + \boldsymbol{\omega} \times \mathbf{u} + 2\boldsymbol{\Omega} \times \mathbf{u} = -\nabla P + \nu \nabla^2 \mathbf{u} + \mathbf{f}, \quad (2.1)$$

where \mathbf{u} is the velocity field, $\boldsymbol{\omega} = \nabla \times \mathbf{u}$ is the vorticity, P is the pressure, ν is the kinematic viscosity, and \mathbf{f} is an external mechanical force. In a Cartesian domain, we choose the rotation axis to be in the z direction with $\boldsymbol{\Omega} = \Omega \mathbf{e}_z$, where Ω is the rotation frequency. In the ideal case of $\nu = 0$ and $\mathbf{f} = 0$, Eq. (2.1) conserves the energy $E = \frac{1}{2} \langle |\mathbf{u}|^2 \rangle$ (where $|\cdot|$ stands for the L_2 -norm) and the helicity $H = \langle \mathbf{u} \cdot \boldsymbol{\omega} \rangle$ with the angular brackets denoting a spatial average unless indicated otherwise.

τ_m/τ_{NL}	0.5	0.5	0.5	0.5	0.5	0.5	4.0	32.0	128.0	∞	∞	∞	∞	∞	∞	∞
Ro_f	0.01	0.05	0.1	0.1	0.2	0.5	0.1	0.1	0.1	0.01	0.05	0.1	0.1	0.2	0.33	0.5
Re_f	333	333	333	714	333	333	333	333	333	333	333	333	714	333	333	333
Ω	50.0	10.0	5.0	5.0	2.5	1.0	5.0	5.0	5.0	50.0	10.0	5.0	5.0	1.0	1.5	1.0
$\nu (\times 10^{-3})$	3.0	3.0	3.0	1.4	3.0	3.0	3.0	3.0	3.0	3.0	3.0	3.0	1.4	3.0	3.0	3.0
N	256	256	256	512	256	256	256	256	256	256	256	256	512	256	256	256

Table 1: Numerical control parameters of the DNS.

The external mechanical forcing in Eq. (2.1) is given by

$$\mathbf{f} = f_0 \sum_{k_f} \begin{pmatrix} \sin(k_f y + \phi_y) + \sin(k_f z + \phi_z) \\ \sin(k_f x + \phi_x) + \sin(k_f z + \phi_z) \\ \sin(k_f x + \phi_x) + \sin(k_f y + \phi_y) \end{pmatrix}. \quad (2.2)$$

For all the runs the forcing amplitude is normalised such that $\mathbf{f}/|\mathbf{f}| = f_0 = 1$ and the phases ϕ_x, ϕ_y, ϕ_z are randomised for the forcing wavenumbers $k_f = 2$ to 4 every τ_m , the so called memory time scale, which is one of the control parameters in our study. In the limit of $\tau_m \rightarrow 0$ we have essentially a random delta correlated in time forcing with the phases randomised at each time step, whereas when we choose $\tau_m = \infty$ we randomise the phases only at $t = 0$ in the duration of the runs and hence we apply a time-independent forcing. Note that our forcing mechanism has $\mathbf{f} \cdot \nabla \times \mathbf{f} \neq 0$ pointwise in space but it is non-helical on average $\langle \mathbf{f} \cdot \nabla \times \mathbf{f} \rangle = 0$.

Now, if we write the wavenumbers in the 3D Fourier space using cylindrical coordinates, we have $\mathbf{k} = (\mathbf{k}_\perp, \mathbf{k}_\parallel)$, with $k_\perp = \sqrt{k_x^2 + k_y^2}$ and $k_\parallel = |k_z|$. Then, the 2D modes (i.e. independent of z) in Fourier space can be denoted as $\mathbf{u}(\mathbf{k}_\perp)$ and the 3D or wave modes as $\mathbf{u}(\mathbf{k})$. Then, in this setting the time-independent forcing ($\tau_m = \infty$) excites two 2D modes in the k_x and k_y axis of the Fourier space and one 3D mode in the k_z axis in the Fourier shell of amplitude $|\mathbf{k}| = k_f$. The random-in-time forcing ($\tau_m \rightarrow 0$) excites also the same modes but since the phases are random, this mechanism is essentially isotropic in contrast to the time-independent forcing.

The relevant dimensionless control parameters of our problem are defined based on the forcing amplitude. So, the Reynolds number is given by $Re_f = U/(k_{min}\nu)$ and the Rossby number by $Ro_f = Uk_{min}/(2\Omega)$ where $U = (f_0/k_{min})^{1/2}$. Using these definitions Re_f^2 is essentially the Grashof number and Ro_f the ratio of the rotation period $\tau_w \propto \Omega^{-1}$ to the eddy turnover time $\tau_{NL} = (Uk_{min})^{-1}$. Note that Re_f and Ro_f are control parameters that they do not require knowledge of the solution to be evaluated and are useful for comparison with body-forced numerical simulations or experiments. All the control parameters of our DNS are listed in Table 1.

Using the pseudo-spectral method, we numerically integrate Eq. (2.1) in a periodic box of size 2π , satisfying the incompressibility condition $\nabla \cdot \mathbf{u} = 0$. The time derivatives are estimated using a third-order Runge-Kutta scheme. Aliasing errors are removed using the 2/3 dealiasing rule and as a result the minimum and maximum wavenumbers are $k_{min} = 1$ and $k_{max} = N/3$, respectively, where N is the number of grid points in each Cartesian coordinate. For more details on the numerical code see (Gómez *et al.* 2005).

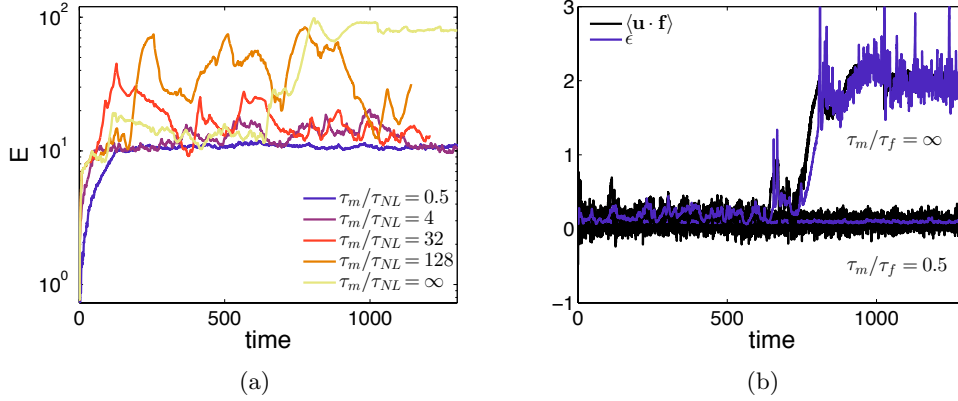


Figure 1: (Color online) Time series of (a) the energy E for the flows with different memory time scales τ_m of the forcing and of (b) the rates of energy dissipation ϵ and injection $\langle \mathbf{u} \cdot \mathbf{f} \rangle$ for the flows with $\tau_m/\tau_{NL} = 0.5$ and ∞ at Rossby number $Ro_f = 0.1$ and Reynolds number $Re_f = 333$

3. Forcing-dependent dynamics

3.1. Time evolution

Figure 1a shows the temporal evolution of the energy E for flows with $Ro_f = 0.1$ and different memory time scale τ_m of the forcing. As τ_m increases, we observe a gradual increase of the amplitude of the energy up to an order of magnitude. The time-series for $\tau_m/\tau_{NL} > 4$ are characterised by large signal variations, which require extremely long time-integrations restricting our runs to moderate Reynolds numbers. Note that even for low τ_m a steady state is reached after a transient that lasts for about 50 to $100\tau_{NL}$ turnover times indicating how expensive computationally is to reach a steady state regime in rotating flows forced at large scales.

The temporal evolution of the energy dissipation rate $\epsilon = \nu \langle |\boldsymbol{\omega}|^2 \rangle$ and the energy injection rate $\langle \mathbf{u} \cdot \mathbf{f} \rangle$ are presented in Fig. 1b. For clear illustration purposes we choose to plot only the two extreme cases of the flows with the highly random-in-time forcing ($\tau_m/\tau_{NL} = 0.5$) and the time-independent forcing ($\tau_m/\tau_{NL} = \infty$) at $Ro_f = 0.1$. As we saw at relatively early times the two flows reach a steady state (see Fig. 1a) and therefore the balance $\epsilon = \langle \mathbf{u} \cdot \mathbf{f} \rangle$ is satisfied with both flows having the same rates of energy injection and dissipation. However, after a very long time period ($\sim 600\tau_{NL}$ turnover times) the flow with the time-independent forcing deviates to a new statistically steady state. This happens when \mathbf{u} becomes correlated with \mathbf{f} and then the flow adjusts its dissipation rate such that a new steady state is achieved.

This adjustment of the dissipation rate by the flow owing to the increase of the correlation between the external mechanical force and the velocity field explains why the energy increases as we increase the memory time scale of the forcing. This is a very interesting property of rotating flows from a practical point of view if one wants to minimise or maximise the energy dissipation rate in a potential application such as in turbomachinery.

3.2. The role of helicity

Helicity is common in real flows and it can be created, for example, in planetary atmospheres in the presence of rotation and stratification (Moffatt 1978; Tobias 2009; Marino

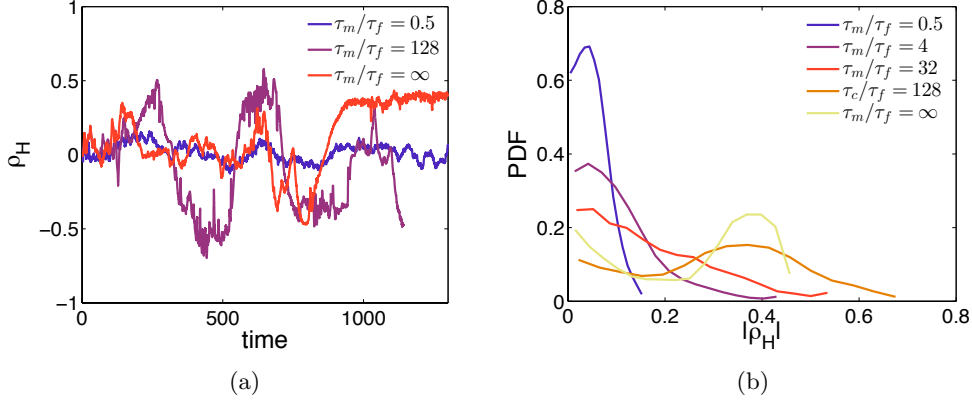


Figure 2: (Color online) (a) Time series of relative helicity ρ_H and (b) Probability Density Function (PDF) of the absolute value of ρ_H for the flows with different forcing memory time scale at Rossby number $Ro_f = 0.1$ and Reynolds number $Re_f = 333$

et al. 2013). In homogenous non-rotating turbulence it is expected that the helicity spectrum cannot develop if it is initially zero (André & Lesieur 1977) or if an external mechanism does not inject net helicity (Dallas *et al.* 2015). In our runs zero net helicity is injected into the flow. Nevertheless, for a given rotation rate (i.e. $Ro_f = 0.1$) we observe that the relative helicity $\rho_H = H/(|\mathbf{u}||\boldsymbol{\omega}|)$ increases as the memory time scale of the forcing increases (see Fig. 2).

It is apparent from Fig. 2 that the mirror-symmetry breaking depends on the value of τ_m . Figure 2a shows ρ_H to be almost zero at early times for all the flows and as τ_m increases the mirror-symmetry breaks at later times only for long enough τ_m . We observe that helicity emerges in the flow as soon as τ_m becomes of the order of the eddy turnover time, i.e. $\tau_m/\tau_{NL} \sim \mathcal{O}(1)$. To analyse further this behaviour of ρ_H we plot the Probability Density Function (PDF) of the time series of the absolute value of relative helicity in Fig. 2b. This plot shows an increase in the mean value of $|\rho_H|$ and also a broadening of the tails of the PDFs for longer memory time scales. So, unlike in homogeneous turbulence, helicity can be created in rotating flows by an external force with a sufficiently long memory time scale, even though net helicity is not injected directly into the flow.

To determine whether the breaking of mirror-symmetry, that distinguishes flows with highly random-in-time and time-independent forcings, remains at higher Reynolds numbers we carried out simulations for the two extreme cases of $\tau_m/\tau_{NL} = 0.5$ and ∞ at $Re_f = 714$ and $Ro_f = 0.1$. Our numerical simulations confirm the persistence of this behaviour at higher Reynolds numbers. We therefore analyse these higher Reynolds number runs in order to gain further insight on the effects of helicity on the flow.

Visualisations of the relative helicity of the flows with $Re_f = 714$ are presented in Figs. 3a and 3b with the value of $\rho_H = -0.018$ and 0.45 for $\tau_m/\tau_{NL} = 0.5$ and ∞ , respectively. The red and blue colour in Fig. 3 indicate right-hand (positive helicity) and left-hand (negative helicity) circularly polarized helical waves, respectively. An instructive way to explain this further is to decompose the velocity field into circularly polarised helical waves (Constantin & Majda 1988; Waleffe 1992)

$$\mathbf{u}(\mathbf{x}, t) = \mathbf{h}_{\pm}(\mathbf{k})e^{i(\mathbf{k} \cdot \mathbf{x} - \omega_{\pm}t)}, \quad (3.1)$$

where $i\mathbf{k}$, \mathbf{h}_+ and \mathbf{h}_- are the linearly independent eigenvectors of the curl operator, i.e. $i\mathbf{k} \times \mathbf{h}_{\pm} = \pm|\mathbf{k}|\mathbf{h}_{\pm}$. These complex eigenvectors are orthogonal to each other and are

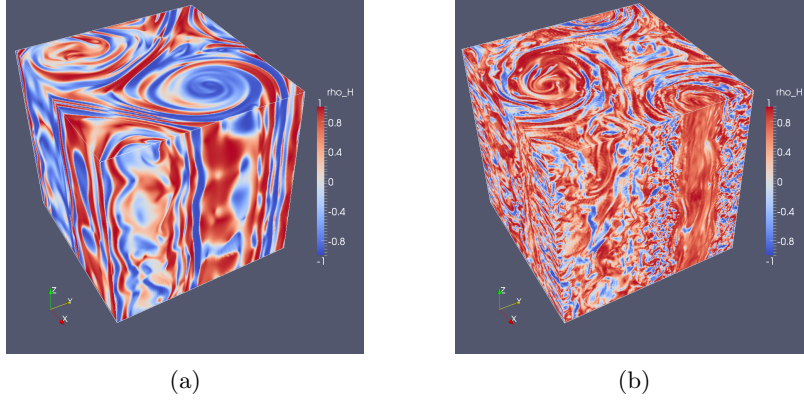


Figure 3: (Color online) Visualisations of the relative helicity ρ_H for the flows at Rossby number $Ro_f = 0.1$ and Reynolds number $Re_f = 714$ with (a) $\tau_m/\tau_{NL} = 0.5$, $\rho_H = -0.018$ and (b) $\tau_m/\tau_{NL} = \infty$, $\rho_H = 0.45$

fully helical. So, now $\hat{\mathbf{u}}(\mathbf{k})$ can be expressed as a linear combination of the eigenvectors \mathbf{h}_+ and \mathbf{h}_- only as follows

$$\hat{\mathbf{u}}(\mathbf{k}, t) = u_+(\mathbf{k}, t)\mathbf{h}_+(\mathbf{k}) + u_-(\mathbf{k}, t)\mathbf{h}_-(\mathbf{k}) \quad (3.2)$$

since $\mathbf{k} \cdot \hat{\mathbf{u}}(\mathbf{k}) = 0$. Then, the helicity can be separated into modes of positive and negative helicity, viz.

$$\begin{aligned} H &= \sum_{\mathbf{k}} \hat{\mathbf{u}}(\mathbf{k}) \cdot \hat{\boldsymbol{\omega}}^*(\mathbf{k}) \\ &= \sum_{\mathbf{k}} k(|u_+(\mathbf{k})|^2 - |u_-(\mathbf{k})|^2) \\ &= k(E_+ - E_-) = H_+ - H_-, \end{aligned} \quad (3.3)$$

where $*$ denotes the complex-conjugate.

The nature of the forcing is clearly imprinted on the flow structure in Fig. 3. The flow with the highly random-in-time forcing (see Fig. 3a) gives a quasi-2D flow with two large columnar vortices, typical at low Rossby numbers due to the Taylor-Proudman theorem. These two vortices are governed by helical waves of opposite polarity. On the other hand, the flow with the time-independent forcing is characterised by helical waves of opposite polarity that travel within the flow breaking the quasi-2D behaviour at small and intermediate scales that is imposed by rotation (see Fig. 3b). Note that the two large scale vortices are still present but this time they have the same sign of helicity on average. Similar effects of helicity have also been observed on an elder study of decaying rotating turbulence (Morinishi *et al.* 2001).

In order to quantify the level of anisotropy of these two runs we consider the 2D energy spectrum which is defined as

$$E_{2D}(k_{\perp}, k_{\parallel}) = \sum_{\substack{k_{\parallel} \leq |\mathbf{k} \cdot \mathbf{e}_z| < k_{\parallel} + 1 \\ k_{\perp} \leq |\mathbf{k} \times \mathbf{e}_z| < k_{\perp} + 1}} |\hat{\mathbf{u}}_{\mathbf{k}}|^2. \quad (3.4)$$

The sum is restricted here at energy in cylinders of radius k_{\perp} and energy in planes k_{\parallel} . Figures 4a and 4b show the 2D energy spectrum for the flows with $\tau_m/\tau_{NL} = 0.5$ and ∞ , respectively. The contours of the 2D energy spectrum for an isotropic flow is represented

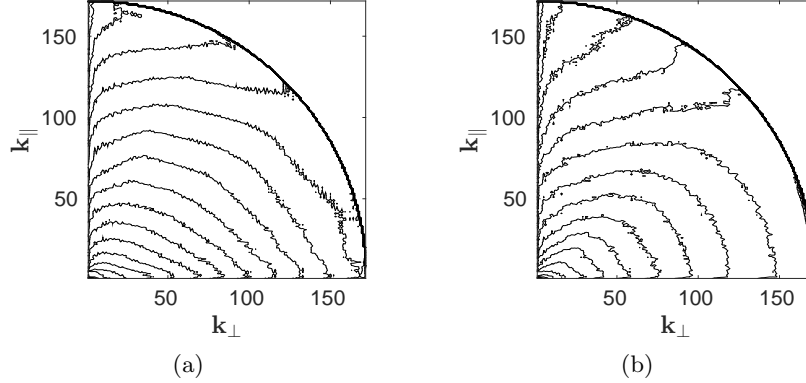


Figure 4: (Color online) The two-dimensional energy spectrum $E_{2D}(k_{\perp}, k_{\parallel})$ for the flows at Rossby number $Ro_f = 0.1$ and Reynolds number $Re_f = 714$ with (a) $\tau_m/\tau_{NL} = 0.5$, $\rho_H = -0.018$ and (b) $\tau_m/\tau_{NL} = \infty$, $\rho_H = 0.45$

by concentric circles centered at the origin of the axes. Any deviation from the circular pattern indicates the level of anisotropy in the flow. By comparing the two contour plots of E_{2D} , it becomes clear that in Fig. 4b the energy is less concentrated close to the $k_{\parallel} = 0$ modes and the intermediate and small scales are closer to isotropy, implying that the flow with the time-independent forcing is overall less anisotropic than the flow with the highly random-in-time forcing. This observation is in agreement with the visualisation of Fig. 3, which prompt us to postulate that the helicity plays a central role on the suppression of anisotropy in the flow.

3.3. Spectral behaviour

In this section we present the spectra of the energy $E(k)$ and the energy flux $\Pi_E(k)$. The energy spectrum was spherically averaged using the following expression

$$E(k) = \sum_{k \leq |\mathbf{k}| < k+1} |\hat{\mathbf{u}}_{\mathbf{k}}|^2 \quad (3.5)$$

and the spectrum of the energy flux was obtained as

$$\Pi_E(k) = \sum_{k=1}^K \sum_{k \leq |\mathbf{k}| < k+1} \hat{\mathbf{u}}^*(\mathbf{k}) \cdot (\widehat{\mathbf{u} \times \boldsymbol{\omega}})(\mathbf{k}). \quad (3.6)$$

The energy flux is a measure that illustrates the direction of the energy cascades. These spectra were time-averaged after the flows have reached a steady state solution. Note that the flow with the time-independent forcing ($\tau_m/\tau_{NL} = \infty$) was time-averaged only after $800\tau_{NL}$ turnover times, when the new steady state was reached (see Fig. 1b).

The effects of the memory time scale of the forcing is also apparent on the spectral dynamics of our flows. Figure 5a shows the energy spectra of the flows with different memory time scales of the forcing compensated by $k^{5/2}$. The spectra of these flows obey different power laws which clearly depend on the memory time scale of the forcing. The runs that are forced with the highly random-in-time forcings have a $k^{-5/2}$ scaling. As τ_m increases the spectra start to deviate gradually from the $k^{-5/2}$ scaling towards a k^{-2} and finally reach a $k^{-5/3}$ scaling for the flow with the time-independent forcing. The $k^{-5/3}$ energy spectrum can be interpreted from the fact that the intermediate and small scales of the flow are closer to isotropy (see Figs. 4b and 3b) and hence we expect

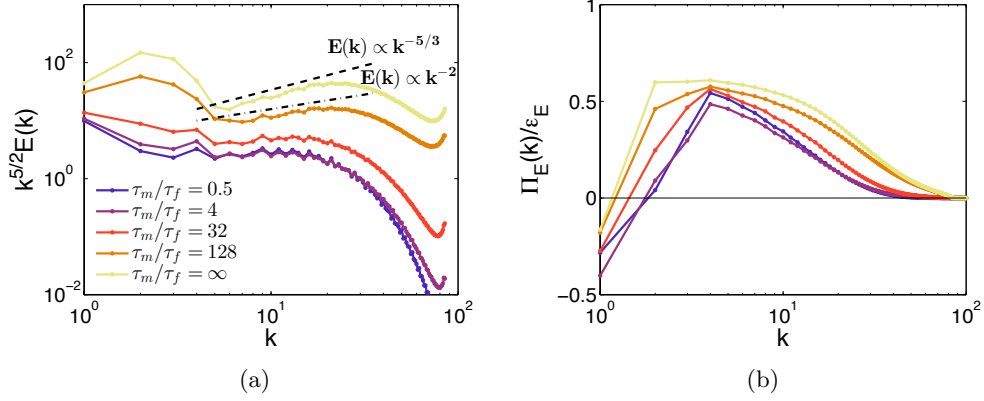


Figure 5: (Color online) (a) Energy spectra $E(k)$ compensated by $k^{5/2}$ and (b) the energy flux spectra $\Pi_E(k)$ normalised with the dissipation rate ϵ_E for flows with different forcing memory time scale at Rossby number $Ro_f = 0.1$ and Reynolds number $Re_f = 333$

the Kolmogorov phenomenology to be valid in this case. All the exponents that we observe here could be related to the various phenomenologies on strong and weak-wave turbulence in the literature, where the interplay between τ_{NL} and the time scale of the inertial waves $\tau_w \propto \Omega^{-1}$ is central to obtain the different energy spectra. These spectral exponents have also been observed in other studies of forced rotating flows (Yeung & Zhou 1998; Mininni & Pouquet 2010; Alexakis 2015). So, it is clear that our results suggest a lack of universality in forced rotating turbulence. However, higher Reynolds number computations integrated for extremely long times are necessary to verify if this is true.

The corresponding spectra for the energy flux $\Pi_E(k)$ normalised by the energy dissipation rate $\epsilon_E = 2\nu \sum_k k^2 E(k)$ are shown in Fig. 5b. The positive flux in this plot indicates a forward cascade while the negative flux indicates a transfer of energy from the small to the large scales of the flow. In the case of negative flux we do not talk about a cascade because we do not have enough scale separation between the forcing scale and the box size. As the memory time scale of the forcing increases, the forward cascade becomes stronger. On the other hand, the flux of energy towards the large scales increases as the forcing becomes more random-in-time with the flow reaching a quasi-2D state. These observations are in line with the visualisations of Fig. 3 which are even at higher Reynolds numbers and thus they favour the non-universal scenario.

We already saw that, as the forcing becomes less time-dependent helicity increases considerably in our flows, so these changes in the spectra can also be related to the presence of strong net helicity in the flow. This is in agreement with prior studies that have shown the influence of helicity on the energy spectrum by directly injecting helicity into the flow (Mininni & Pouquet 2009, 2010). In contrast, helicity does not seem to have any significant effect on the spectra in non-rotating, homogeneous and isotropic helical turbulence (Dallas *et al.* 2015).

Since helicity is not a sign-definite quantity and because we do not inject any net helicity, the sign of helicity in our flows undergoes changes in its inertial range. Therefore, there is either no power-law or difficult to define one in our helicity spectra. For this reason, we do not show any helicity spectra here. In the next section, we examine the Rossby number dependence of the dynamics of the flows.

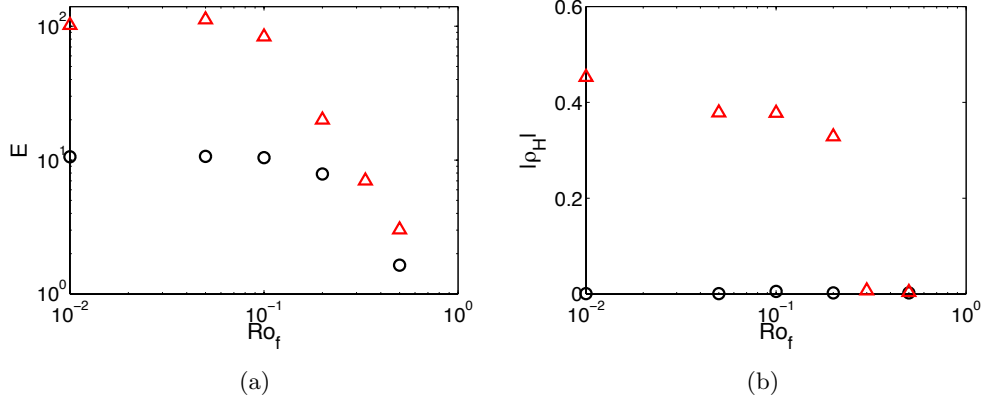


Figure 6: (Color online) Rossby number dependence of (a) energy and (b) absolute value of relative helicity for flows with Reynolds number $Re_f = 333$. The \circ and \triangle denote runs forced with $\tau_m/\tau_{NL} = 0.5$ and $\tau_m/\tau_{NL} = \infty$, respectively.

4. Rossby number dependence

4.1. Global behaviour

In the previous sections we saw that the dynamics depend on the nature of the forcing for a given Rossby number. Here, we investigate the effects of the rotation rate on the flows, focusing on the extreme cases of the forcing being highly random-in-time ($\tau_m/\tau_{NL} = 0.5$) and time-independent ($\tau_m/\tau_{NL} = \infty$) for fixed $Re_f = 333$. Again here we restrict ourselves to moderate Reynolds numbers because extremely long integration times for the runs with time-independent forcing are inevitable.

For high Rossby number flows the effect of rotation is negligible and the energy is expected to flow to scales smaller than the forcing scale. However, as Rossby number is decreased and the flow tends to become quasi-2D, there is more and more energy transferred to scales larger than the forcing scale due to an inverse cascade (Pouquet *et al.* 2013).

We examine the energy and the relative helicity for runs with different Rossby numbers. The triangles and circles denote runs forced with a time-independent forcing and random-in-time forcing, respectively. As Rossby number is decreased we see that energy increases as expected (see Fig. 6a). However, the rate of increase and the values of energy for high enough rotation rates depend on the nature of the mechanical force. Note that the flow with the time-independent forcing has much more energy at small Rossby numbers.

The relative helicity behaves also very differently for the two types of flows and this is shown in Fig. 6b. The flow with the random-in-time forcing has zero net helicity for all Ro_f . However, the flow with the time-independent forcing bifurcates to a state of non-zero helicity for small enough Rossby numbers. The value of $|\rho_H|$ seems to vary discontinuously as Ro_f is decreased with the flow bifurcating to a helical state at the critical $Ro_f^{crit} \simeq 0.2$. Thus, the transition from the non-helical to the helical state is a jump bifurcation. In summary, net helicity emerges in the flow only for small enough Ro_f and long enough τ_m .

Helicity is a pseudoscalar quantity and $H \neq 0$ only if it is directly injected into the flow (i.e. $\mathbf{u} \cdot (\nabla \times \mathbf{f}) \neq 0$) by a helical mechanical force or if another pseudoscalar quantity exists related to the pseudovector $\nabla \times \mathbf{f}$. In our work, we observe that net helicity emerges in rapidly rotating flows with long enough memory time scale forcings only. So,

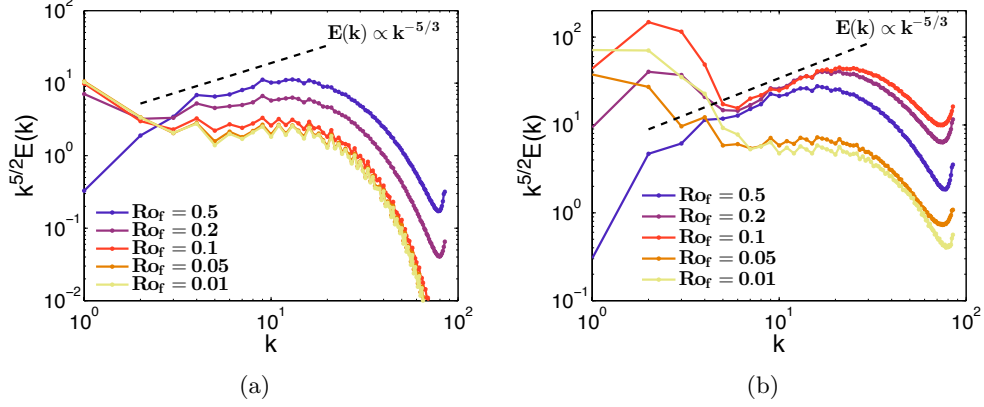


Figure 7: (Color online) Energy spectra $E(k)$ compensated by $k^{5/2}$ for the flows with (a) $\tau_m/\tau_{NL} = 0.5$ and (b) $\tau_m/\tau_{NL} = \infty$ at different Rossby numbers and Reynolds number $Re_f = 333$

a pseudovector that relates the rotation vector with the forcing is $\mathbf{\Omega} \times (\nabla \times \mathbf{f})$ and hence the pseudoscalar quantity that will allow the generation of helicity in a rotating flow is

$$H \propto \mathbf{u} \cdot \mathbf{\Omega} \times (\nabla \times \mathbf{f}). \quad (4.1)$$

A similar expression was derived in a different way by Hide (1975) for a rapidly rotating flow in geostrophic balance assuming that the non-linear term is negligible. Now, from Eq. (4.1) we can deduce that no net helicity will be generated for a short memory time scale forcing since $\langle \nabla \times \mathbf{f} \rangle_t = 0$ (with $\langle \cdot \rangle_t$ denoting an average over time), assuming isotropy and ergodicity. On the other hand, for a forcing with long enough memory time scale $\langle \nabla \times \mathbf{f} \rangle_t = \mathbf{g}(\mathbf{x}) \neq 0$ and therefore $H \neq 0$ for long enough integration time scales in agreement with our observations.

Moffatt (1970) suggested that a random superposition of inertial waves will exhibit a lack of mirror-symmetry if and only if there is a mechanical excitation on a preferred direction in the propagation of the waves with respect to the axis of rotation. Otherwise, the random superposition of inertial waves in equal proportions would give zero net helicity. Based on Eq. (4.1), we conjecture that such a mechanism is pertinent to our flows where the angle φ between $\mathbf{\Omega}e_z$ and $\nabla \times \mathbf{f}$ is fixed at time $t = 0$ for a time-independent forcing and hence such a forcing can add a preferred direction of propagation to the inertial waves inducing the mirror-symmetry breaking in our flows. From the other side, a highly random-in-time forcing can excite inertial waves on all directions in equal proportions since φ is random in time and this is why the net helicity remains zero for any value of Ro_f in this case.

4.2. Spectral behaviour

Here we analyse the energy spectra of the flows at different Rossby numbers. In Fig. 7a we present the energy spectra $E(k)$ compensated by $k^{5/2}$ of the flows with the highly random-in-time forcing. For $Ro_f = 0.5$ the energy spectrum is close to the Kolmogorov $k^{-5/3}$ scaling with the effects of the Coriolis force having no significant influence on the dynamics of the flow. However, as the Rossby number decreases $\tau_w \propto \Omega^{-1}$ becomes the dominant timescale and then the spectrum is changed to the weak-wave turbulence prediction of $E(k) \propto k^{-5/2}$.

Similar behaviour is observed for the spectra of the flows with the time-independent

forcing (see Fig. 7b) but with two different characteristics. The first is the significant condensation of energy at large scales for small enough Rossby numbers in comparison to the flows with the random-in-time forcing. The second is the transition from the Kolmogorov-like regime with $\tau_E \ll \tau_w$ to the weak-wave turbulence regime with $\tau_w \ll \tau_E$, which occurs at lower Rossby numbers, showing the dependence of this transition to the nature of the mechanical force.

Here, we should point out that weak-wave turbulence theory arguments, which assume uniform and isotropic forcing, predict a $k^{-5/2}$ spectrum but they do not predict condensation of energy at large scales due to an inverse cascade in unbounded domains. This is in agreement with Fig. 7a where there is some energy condensation at large scales but not significant in comparison to Fig. 7b. However, the energy condensation at large scales of Fig. 7b suggests that weak-wave turbulence theory is not necessarily valid for the small Rossby number flows with time-independent forcing even though $E(k) \propto k^{-5/2}$.

5. Discussion & conclusions

The dependence of the dynamics of rotating turbulence on the nature of the large scale mechanical force is studied by means of numerical simulations to shed light on the disparate results in the literature. For moderate Reynolds and low Rossby number flows we systematically vary the memory time scale τ_m of the mechanical force. As τ_m increases the forcing mechanism becomes less time-dependent and essentially less isotropic. We are able to demonstrate that different steady state solutions will be reached if one is able to integrate for long enough time scales, showing the dependence of the flows on the forcing mechanism. When $\tau_m \propto \tau_{NL}$ we observe that mirror-symmetry spontaneously breaks in the flow even though our mechanical force is non-helical. Moreover, as the forcing mechanism becomes less time-dependent (long τ_m) the net helicity increases. This is also true for the highest Reynolds number simulations that we carried out. We notice that helical waves break the tendency of the small and intermediate scales of the flows with the time-independent forcing to become 2D due to the imposed strong rotation. This makes the flow less anisotropic in contrast to a flow with highly random-in-time forcing where the net helicity appears to be negligible.

In addition, for moderate Re_f and low Ro_f flows both the power laws for the energy spectrum and the forward and inverse fluxes of energy depend strongly on the forcing mechanism. Depending on the value of τ_m we obtain $E(k) \propto k^{-5/3}$, k^{-2} and $k^{-5/2}$ suggesting a lack of universality in rotating turbulence. Alexakis (2015) showed that no matter how large the Reynolds number can be there is a small enough Rossby number such that the flow exhibits a particular behaviour (e.g. weakly rotating turbulence, quasi-2D condensates) provided that an appropriate $\alpha > 0$ is considered in the scaling $Ro_f \propto Re_f^{-\alpha}$ (where α is expected to depend on the mechanical force). So, it seems plausible that forced rotating turbulent flows are non-universal. To corroborate this argument a large extent of the control parameters space should be covered with higher Reynolds number simulations integrated for extremely long times. However, this is beyond the reach of current computational capabilities.

The Rossby number dependence on the dynamics of flows with a highly random-in-time and a time-independent mechanical force is also investigated at moderate Reynolds numbers. For weakly rotating turbulence (high Ro_f) the total energies of the two systems are comparable. Even so for small enough Ro_f , even though large scale vortices are present in both systems, energy condensates at large scales only for the flow with the time-independent forcing as the energy spectra demonstrate.

Moreover, for large Ro_f the net helicities of the two systems are zero but as Ro_f

becomes smaller there is a critical Ro_f^{crit} which the flow with the time-independent forcing bifurcates discontinuously from a non-helical state to a helical state. On the other hand, the helicity of the flow with the random-in-time forcing remains zero for all values of Ro_f . Based on this observation we argue that the angle between $\Omega \mathbf{e}_z$ and $\nabla \times \mathbf{f}$ is important on the excitation of the inertial waves and consequently on the generation of net helicity in rotating flows. Thus, a time-independent forcing adds a preferred direction of propagation to the inertial waves inducing the mirror-symmetry breaking in our flows, since this angle is fixed in time. From the other side, a highly random-in-time forcing with excites inertial waves on all directions in equal proportions and this is why the net helicity remains zero for any value of Ro_f . Such a mechanism has also been proposed for planetary dynamos (Moffatt 1970).

In the end, the lack of consistency of the results in the literature is attributed here on the forcing-depend dynamics of forced rotating turbulent flows. Experiments should be able to show if this is true at higher Reynolds and lower Rossby numbers. The spontaneous emergence of helicity in such flows is an important aspect with implications on cyclones persistence and intensity in supercell thunderstorms, a phenomenon which defies weather forecasting (Markowski *et al.* 1998) but also in planetary dynamos.

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